Source localization with: Minimum norm estimates (MNE)

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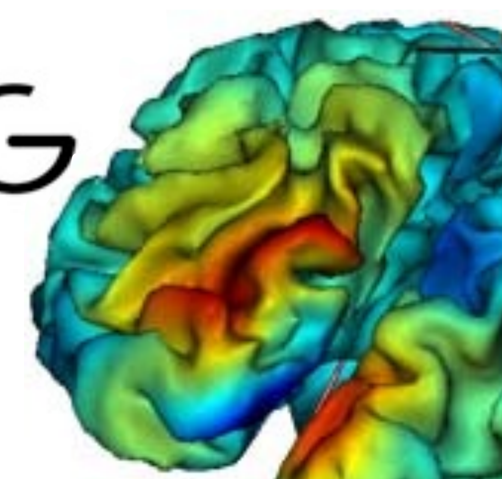
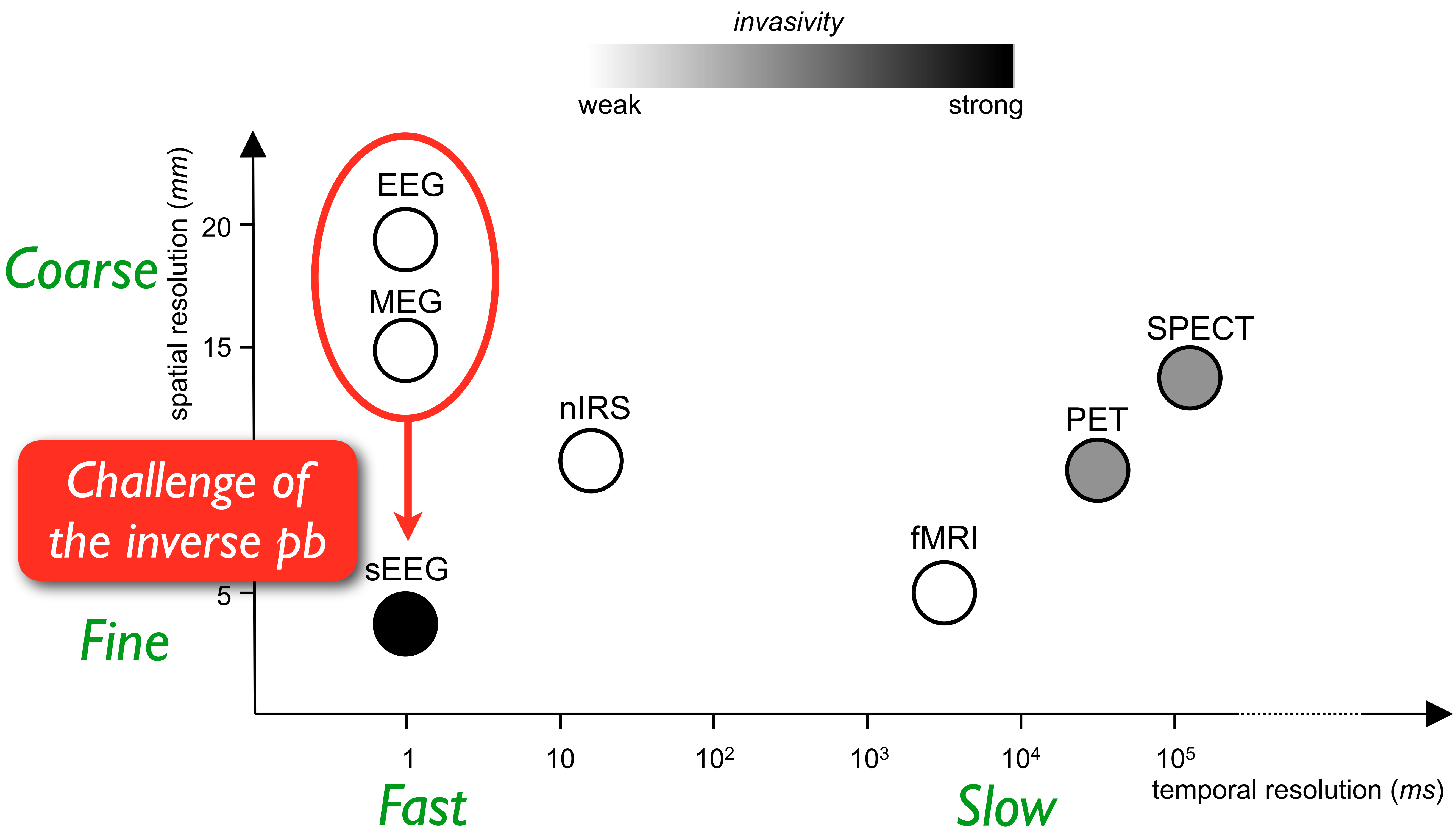


Natmeg - Jan. 2014

Relevant background on M/EEG

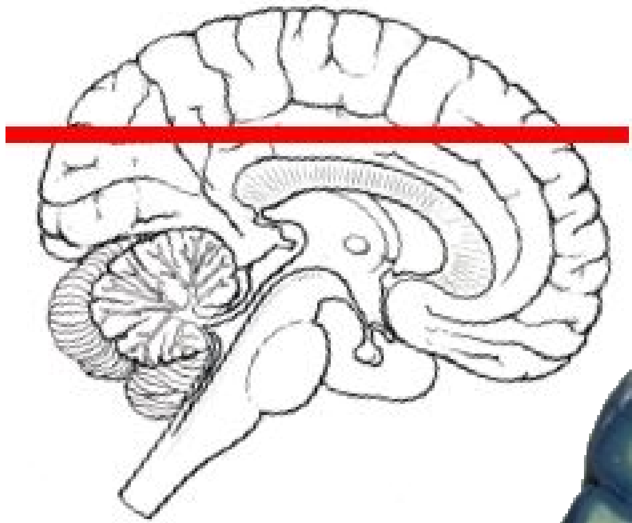
THM: Take home message (not Theorem)...

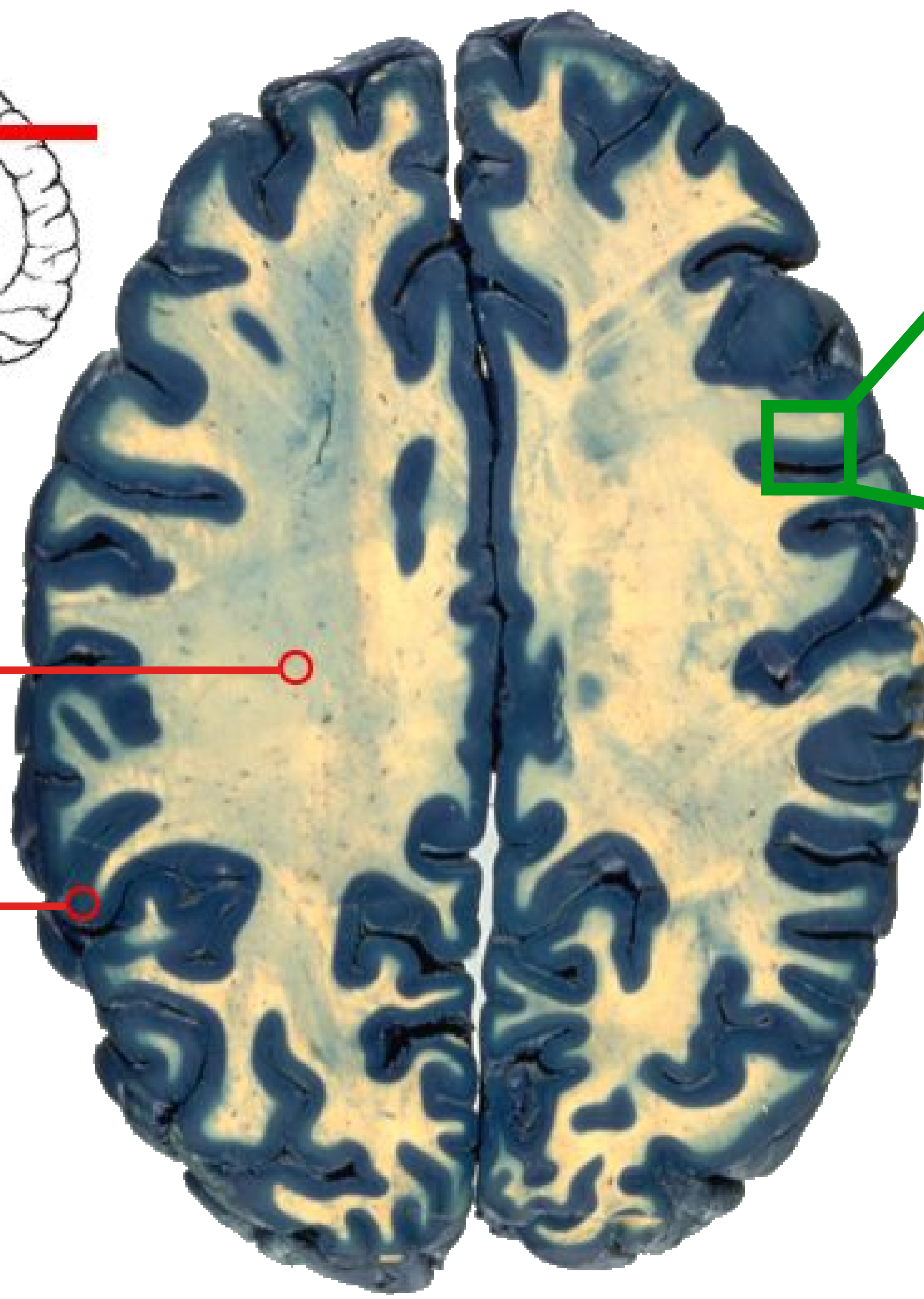
Functional neuroimaging



Brain anatomy

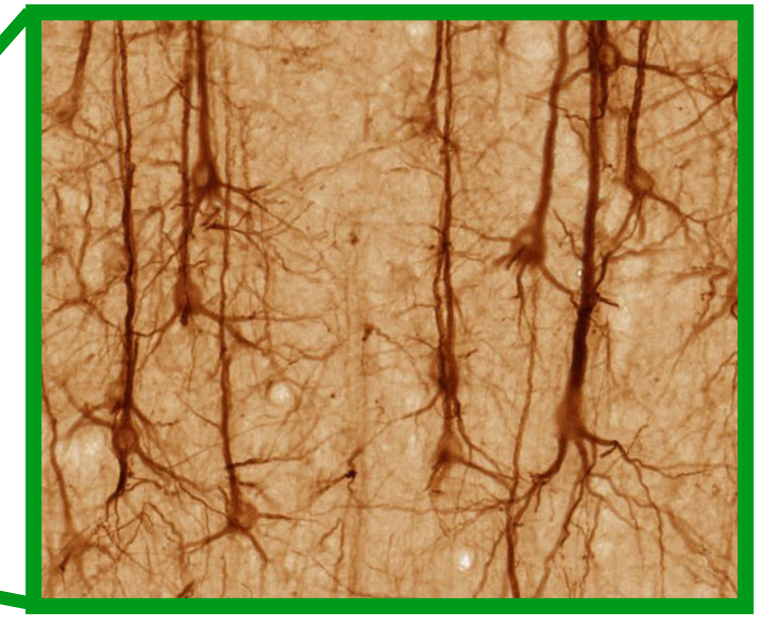
Axial slice





White matter

Gray matter



Neurons

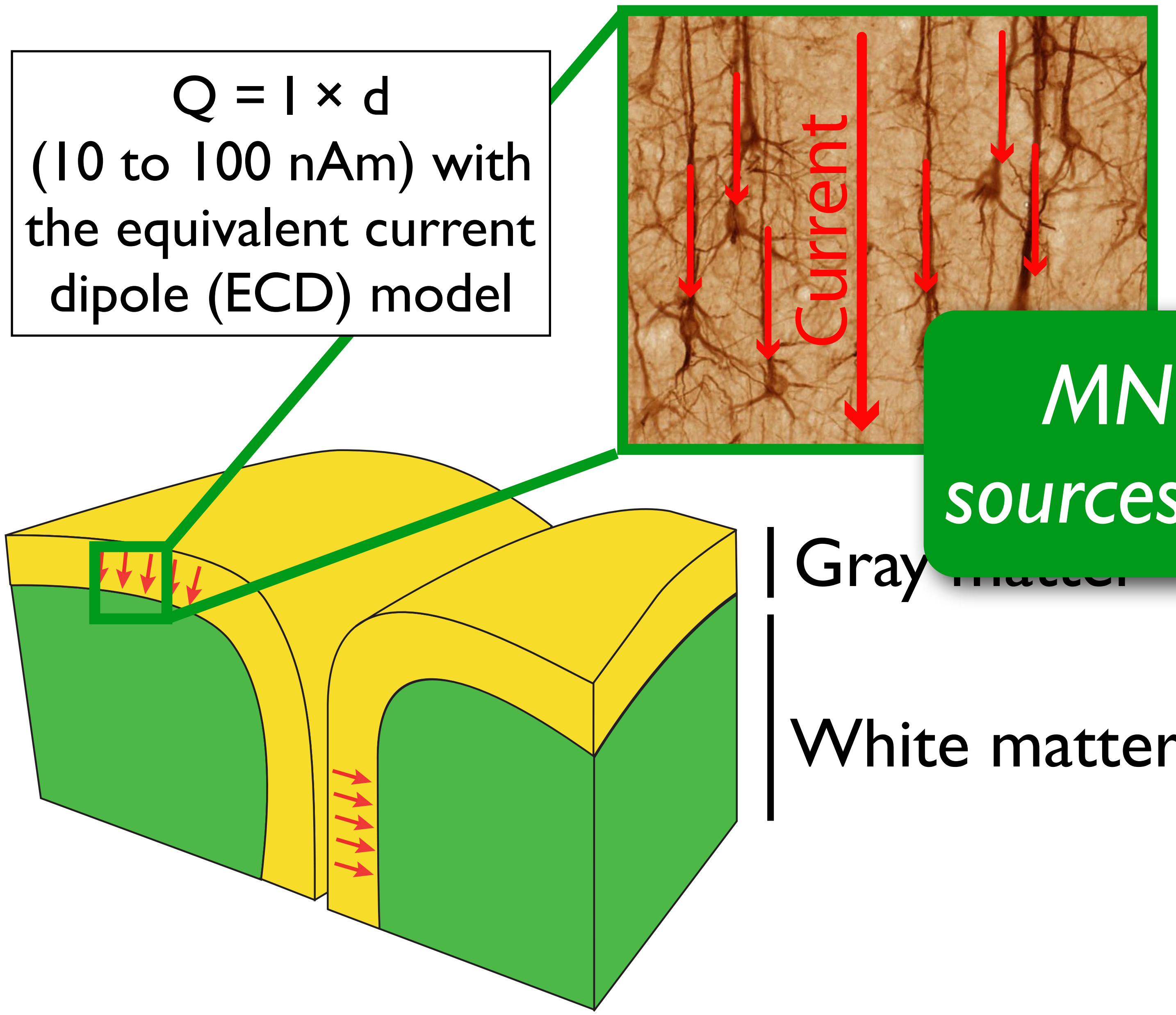
in the gray matter

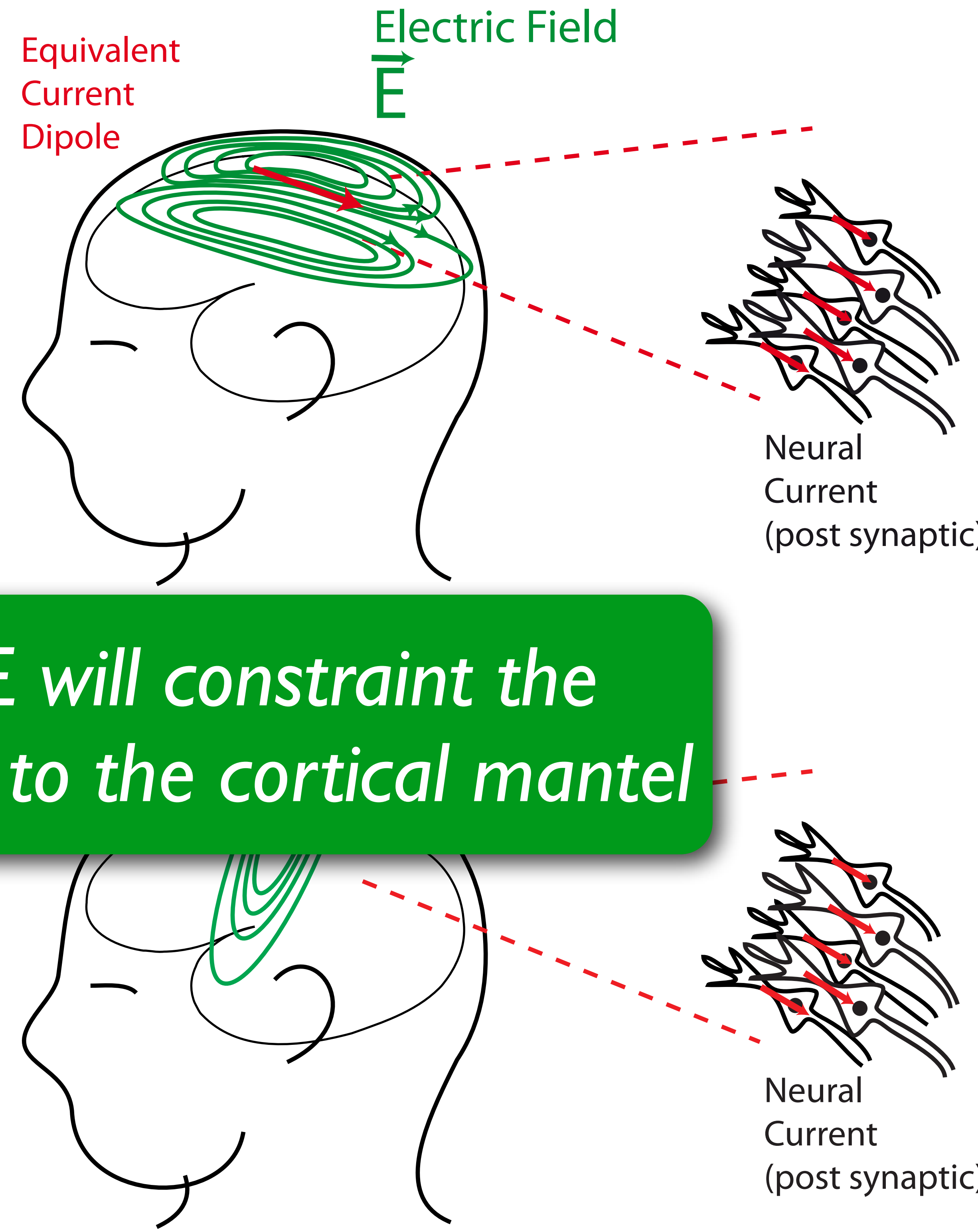
Source: dartmouth.edu

Neurons as current generators

Large cortical pyramidal cells organized in macro-assemblies with their **dendrites normally oriented to the local cortical surface**

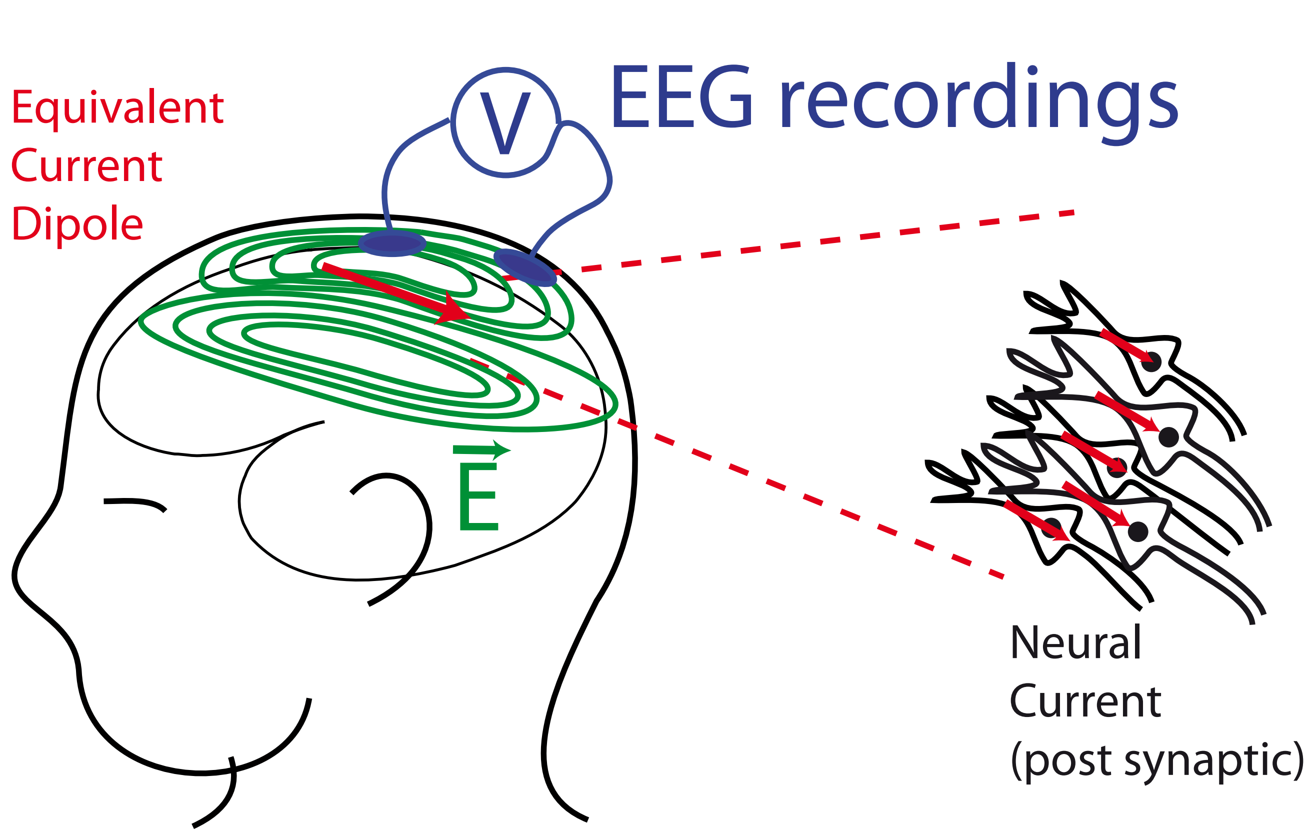
$Q = I \times d$
(10 to 100 nAm) with the equivalent current dipole (ECD) model

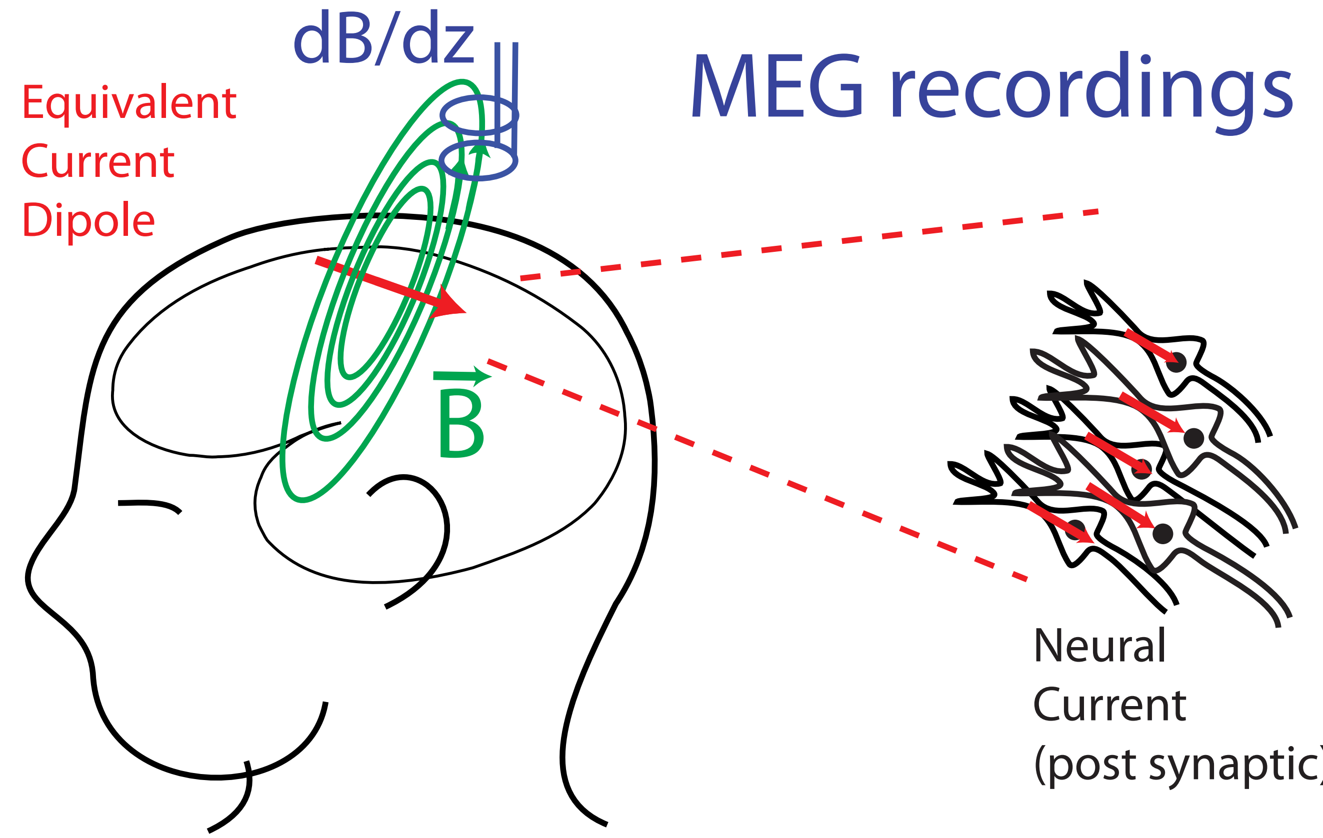




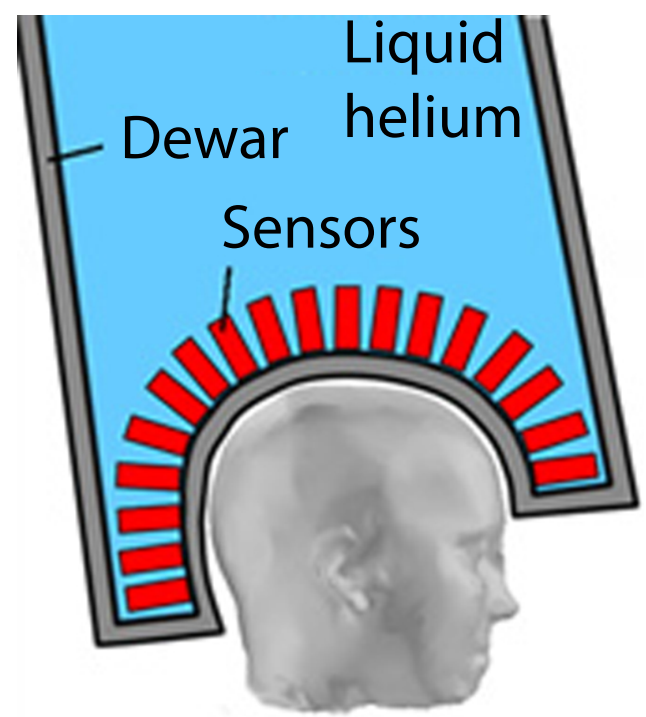
MNE will constraint the sources to the cortical mantel

EEG & MEG systems



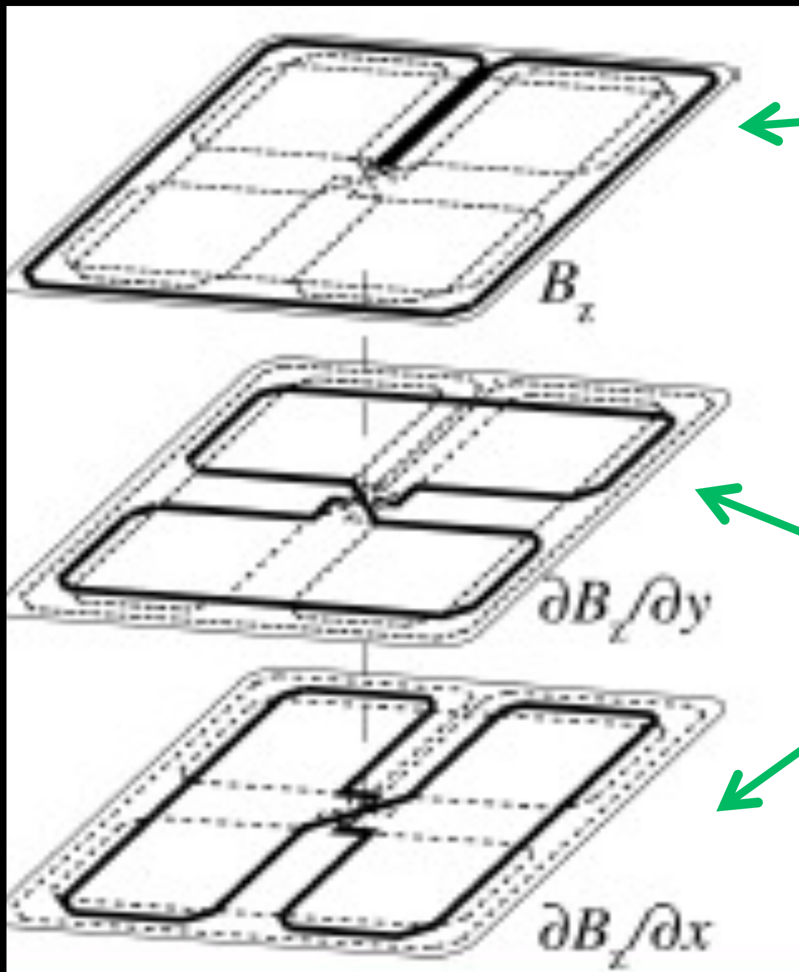


First EEG recordings in 1929 by H. Berger



Hôpital La Timone
Marseille, France

MEG sensors



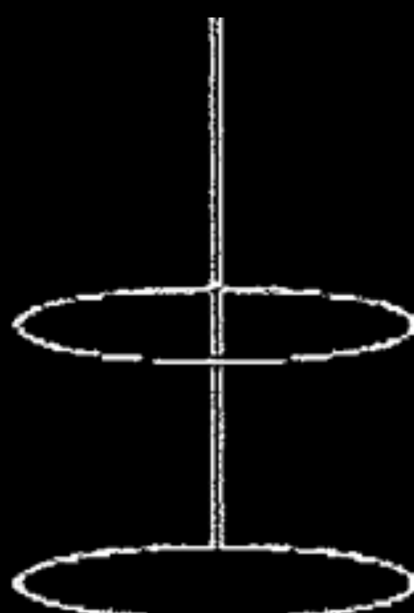
Magnetometer

- General magnetic fields
- Very sensitive overall, **noisy**

Planar Gradiometer

- Focal magnetic fields
- Most sensitive to fields directly underneath

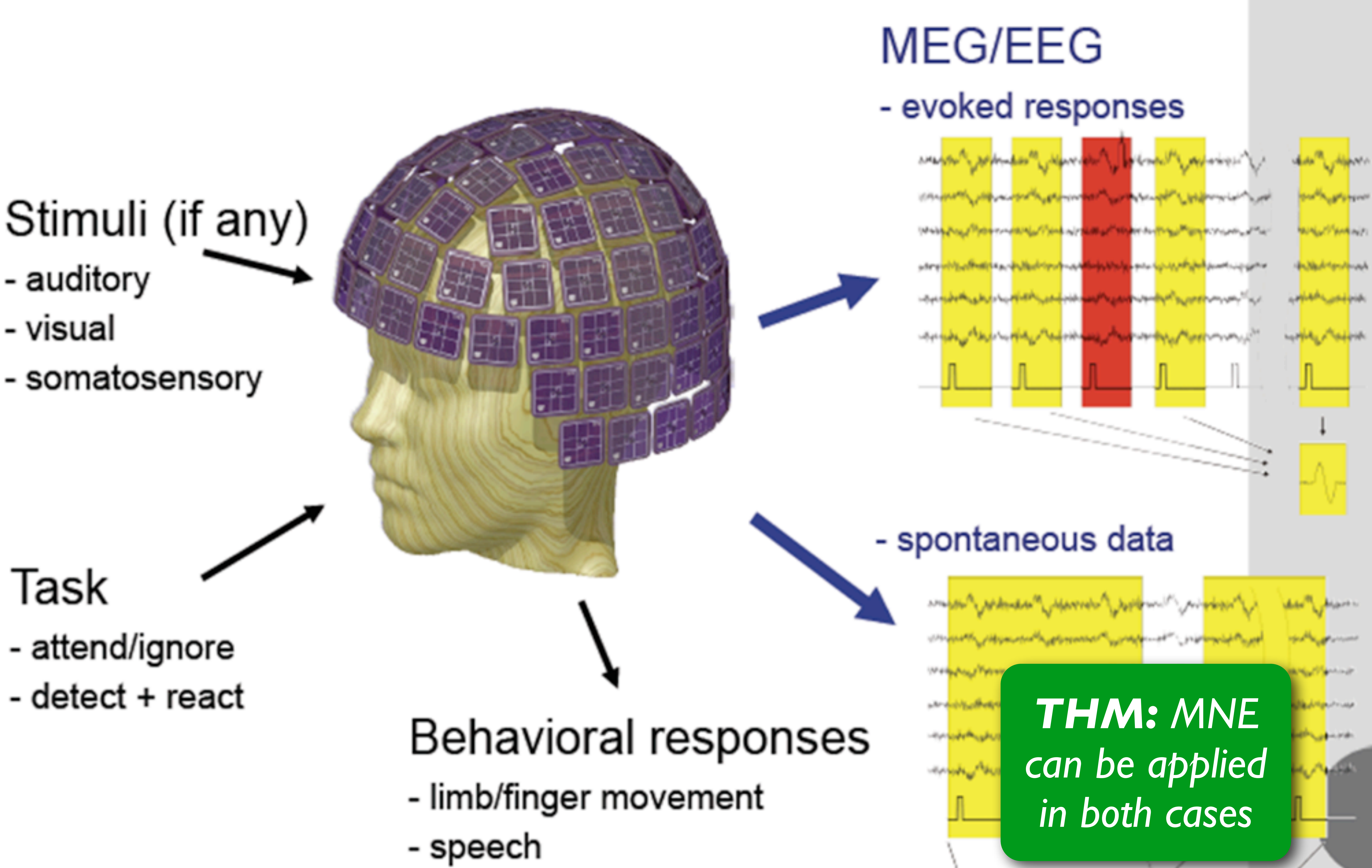
THM: Mix Different Quantities/Units



Axial Gradiometer

- Focal magnetic fields
- Most sensitive to fields directly underneath it

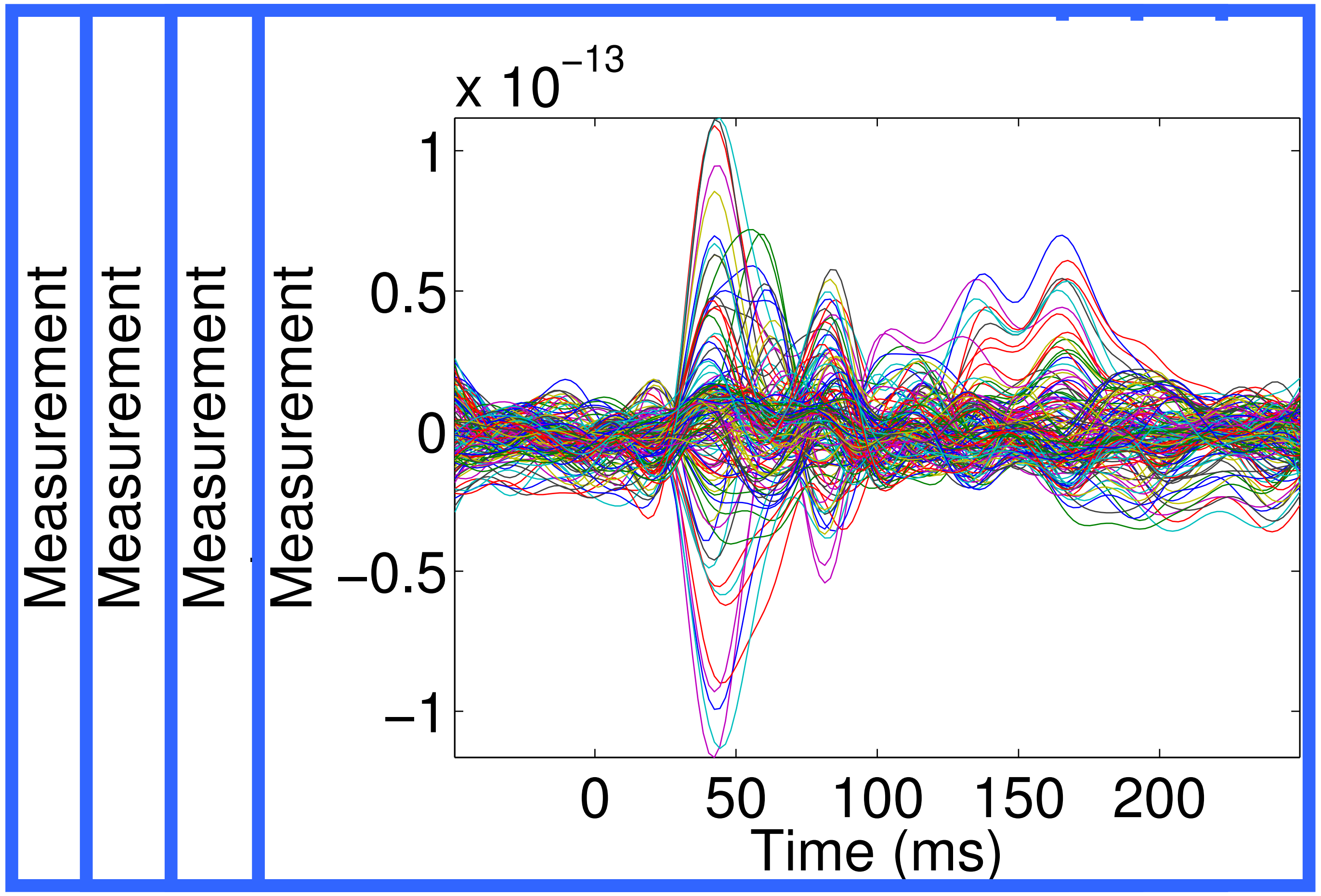
Data acquisition examples



THM: *MNE can be applied in both cases*

Evoked response

Example of trial averaging:



Signal on 151 MEG Channels

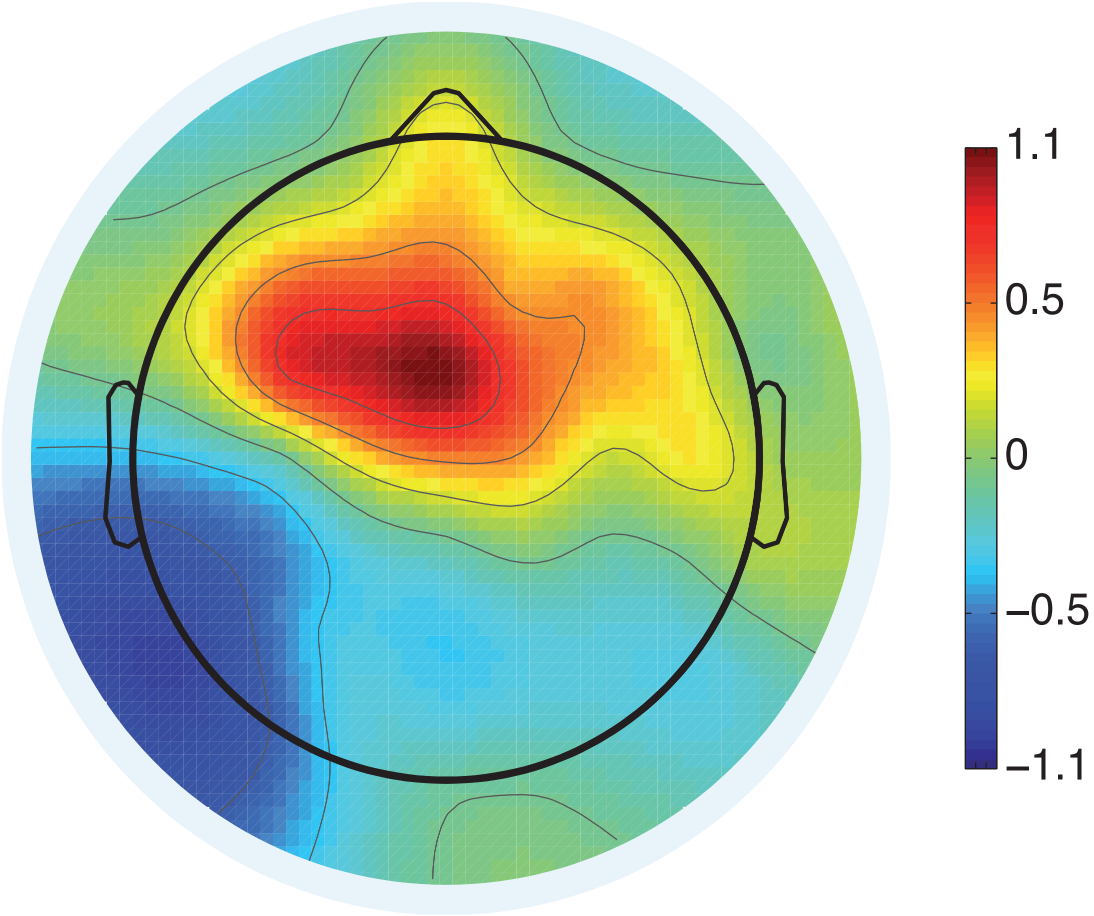
400 trials

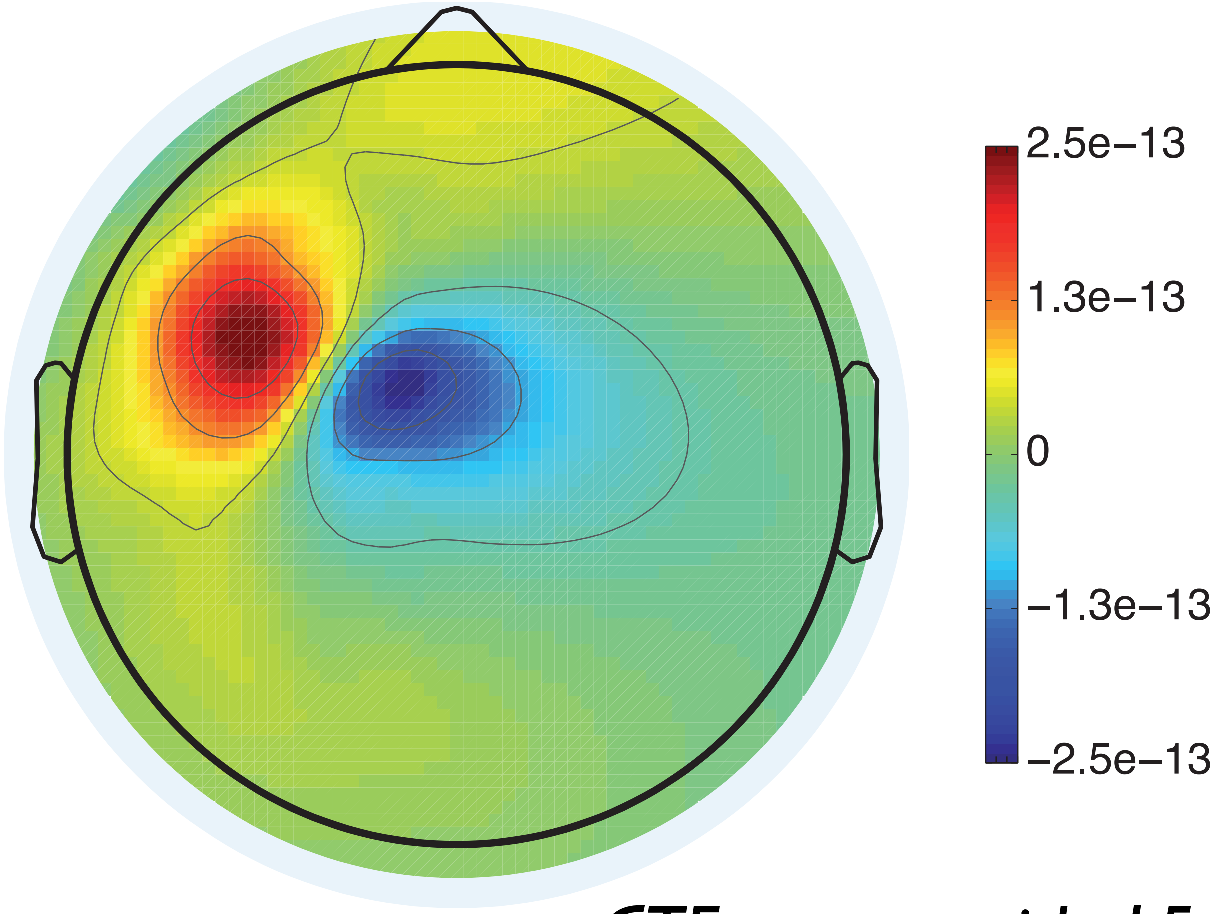
Clean M/EEG data can be obtained by **averaging multiple repetitions** (a.k.a. trials)

M/EEG Measurements

At **one time instant**:

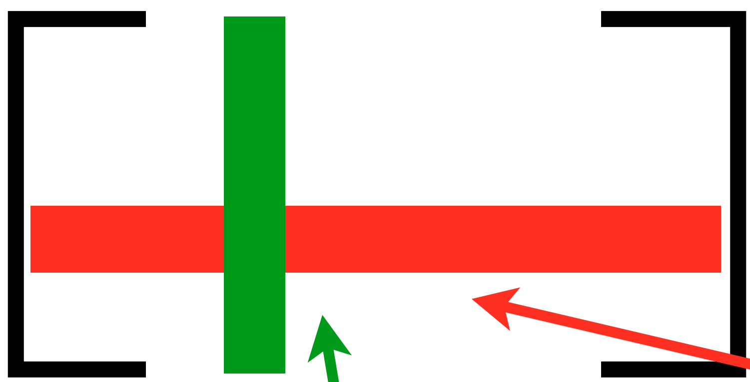
EEG topography vs. MEG topography





CTF system with 151 axial gradiometers

M/EEG Measurements: Notation

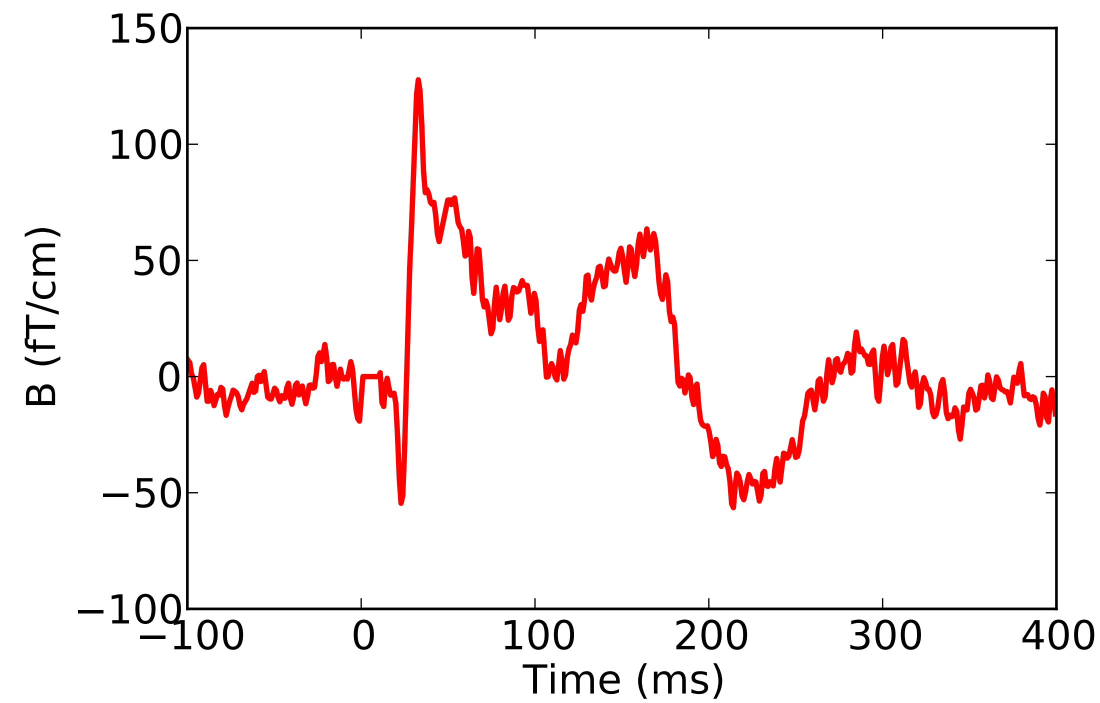
M =  $\in \mathbb{R}^{d_m \times d_t}$

MEG
and/or
EEG

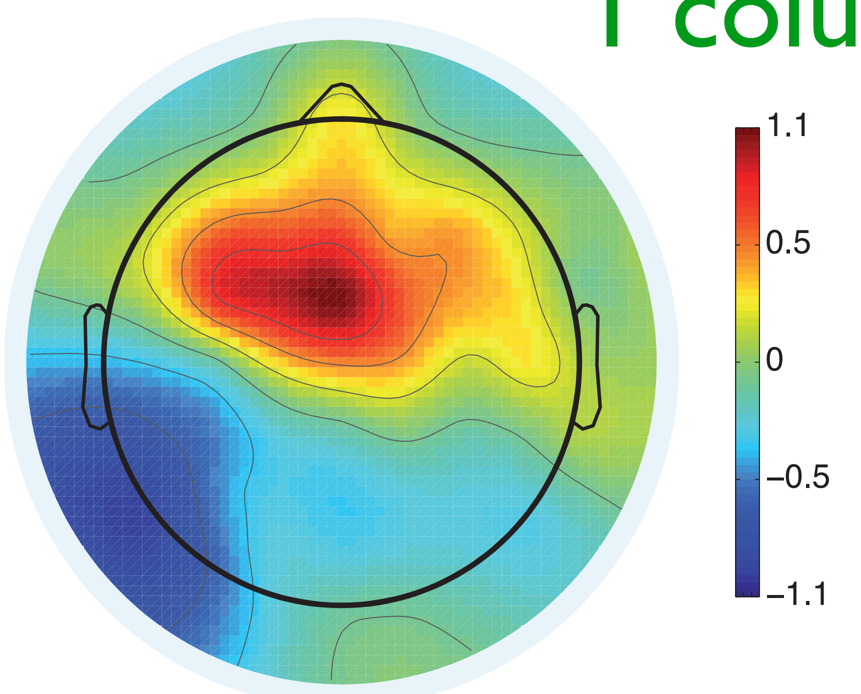
d_m : Nb of sensors

d_t : Nb of time points

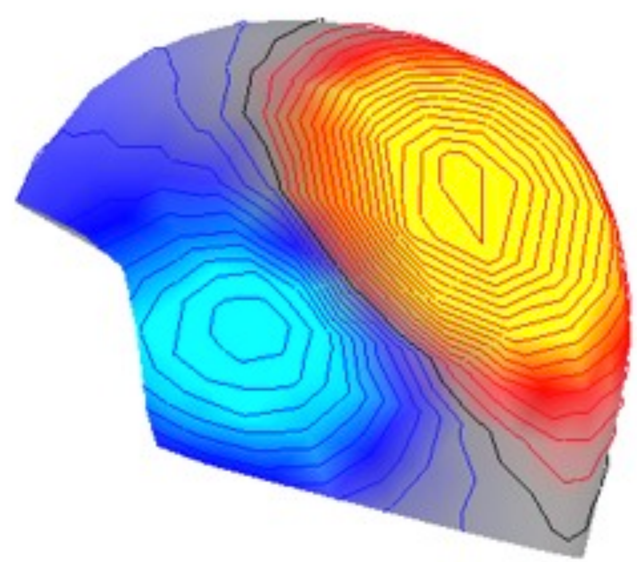
1 column



1 row = 1 time series
on 1 sensor



2D topography

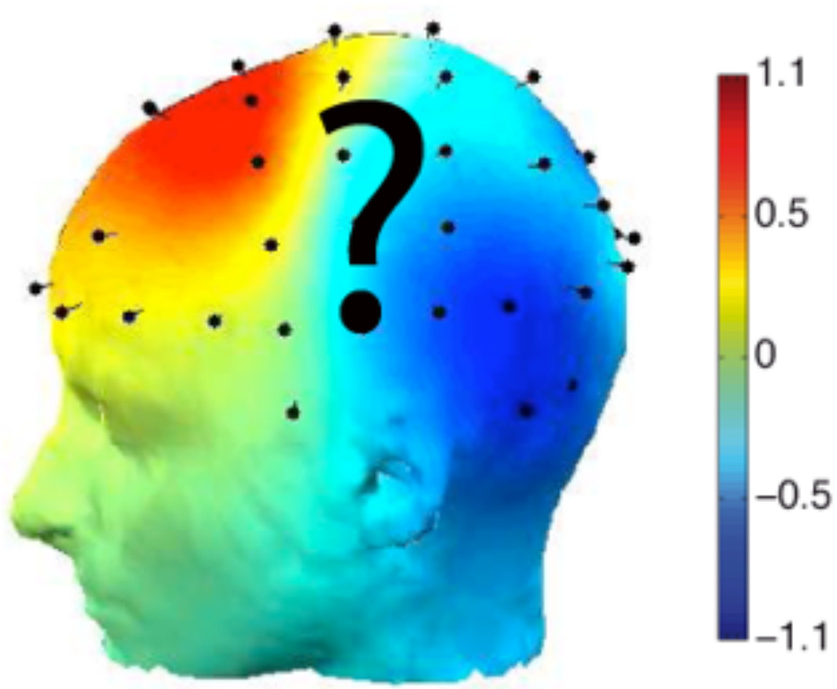


3D topography

The M/EEG inverse problem

Inverse problem: Objective

Find the current generators that produced the M/EEG measurements



Linear forward problem: Maxwell

Maxwell Equations with **quasi-static** approximation

$$\begin{cases} \nabla \times \vec{E} = & 0 \\ \nabla \cdot \vec{B} = & 0 \\ \nabla \times \vec{B} = & \mu_0 \vec{J} \\ \nabla \cdot \vec{E} = & \frac{\rho}{\epsilon_0} \end{cases}$$

Remark: quasi-static implies no temporal derivatives and no propagation delay

Total currents: $\vec{J} = \vec{J}_p + \vec{J}_c$

Primary currents

Conduction currents

Ohm's law: $\vec{J}_c = -\sigma \nabla V$

V Electric potential

σ Tissue conductivity

Potential equation:

(relation btw. the potential and the sources)

$$\nabla \cdot \nabla \times \vec{B} = 0 \Rightarrow \nabla \cdot (\vec{J}_s + \vec{J}_c) = 0$$

$$\Rightarrow \nabla \cdot \vec{J}_p = \nabla \cdot (\sigma \nabla V)$$

THM: Instantaneous & Linear

Head models

Requires to **model the properties of the different tissues**: skin, skull, brain etc.

Hypothesis: The conductivities are **piecewise constant**

Sphere models

Analytical solutions fast to compute but very **coarse** head model (esp. for EEG)

EEG : [Berg et al. 94, De Munck 93, Zhang 95]
MEG : [Sarvas 87]

Realistic models

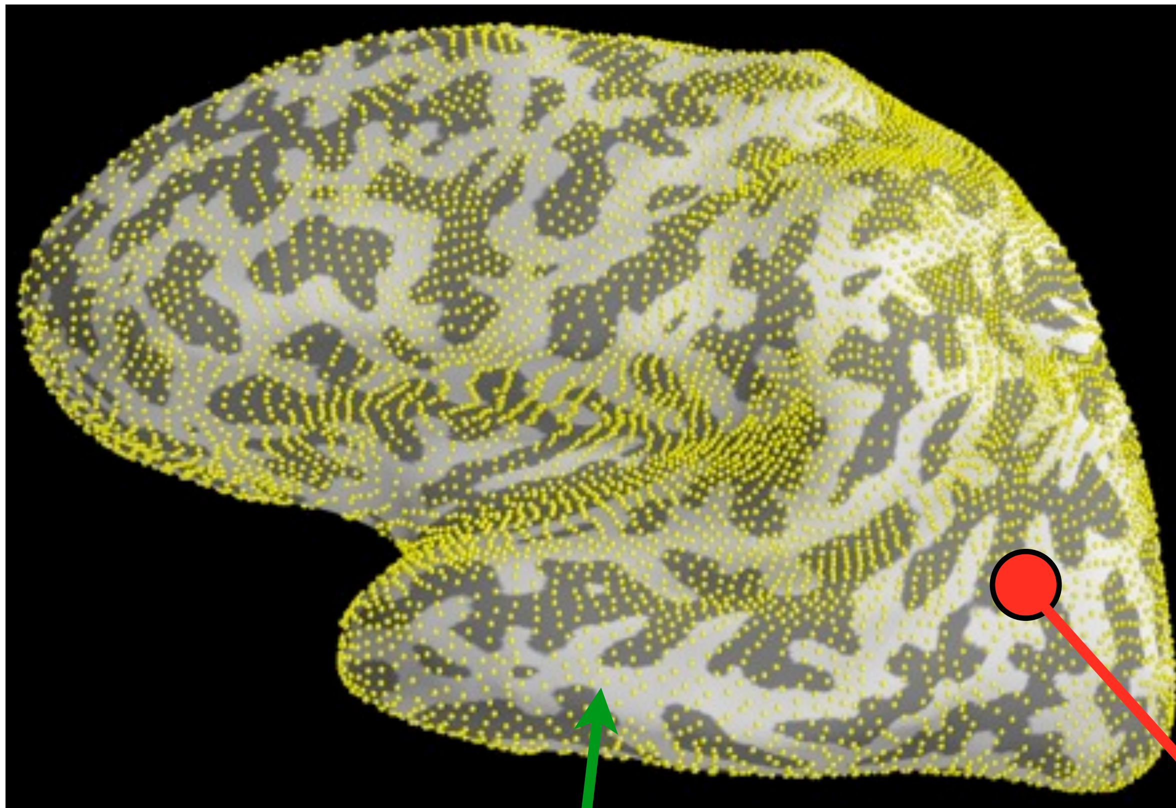
Boundary element method (BEM), i.e., numerical solver with approximate solution.

[Geselowitz 67, De Munck 92, Kybic et al. 2005, Gramfort et al. 2010]

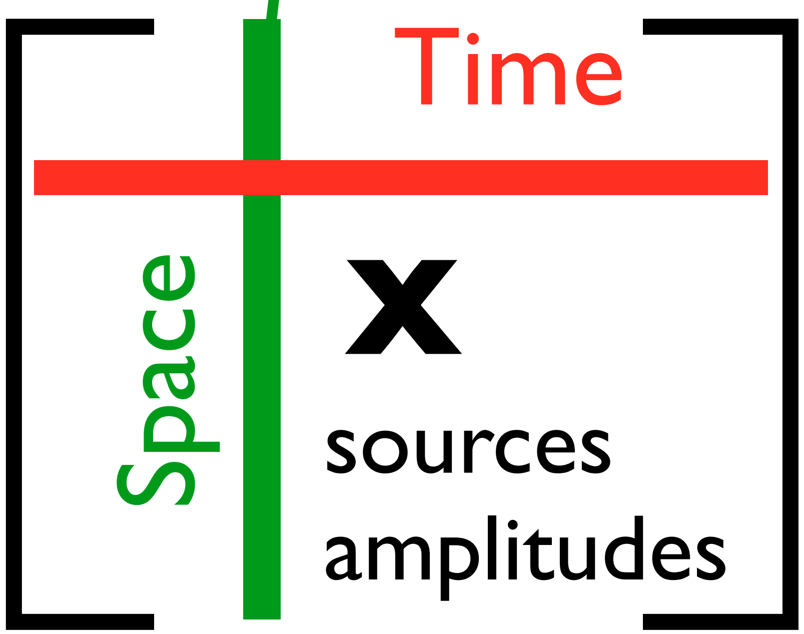
Inverse problem approaches

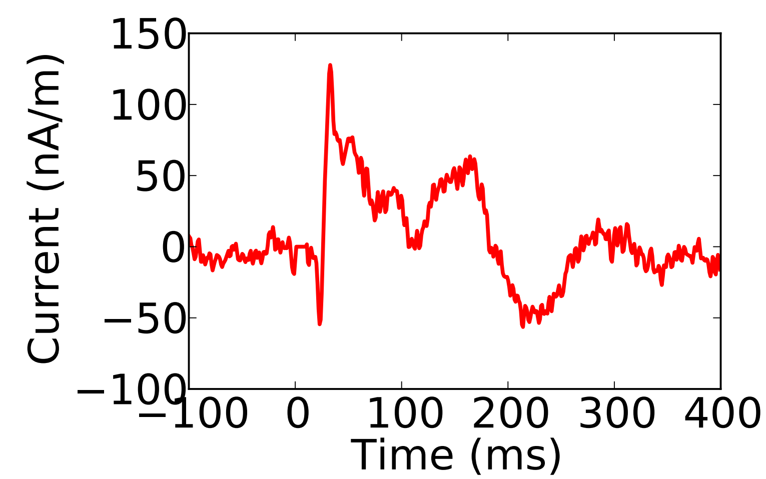
- Dipole fitting
- Scanning methods (Beamformers LCMV, DICS, SAM, MUSIC, RAP-MUSIC)
- Imaging methods with distributed models (MNE, dSPM, sLORETA, LORETA, MxNE, Gamma-Map/Champagne etc…)

Distributed model



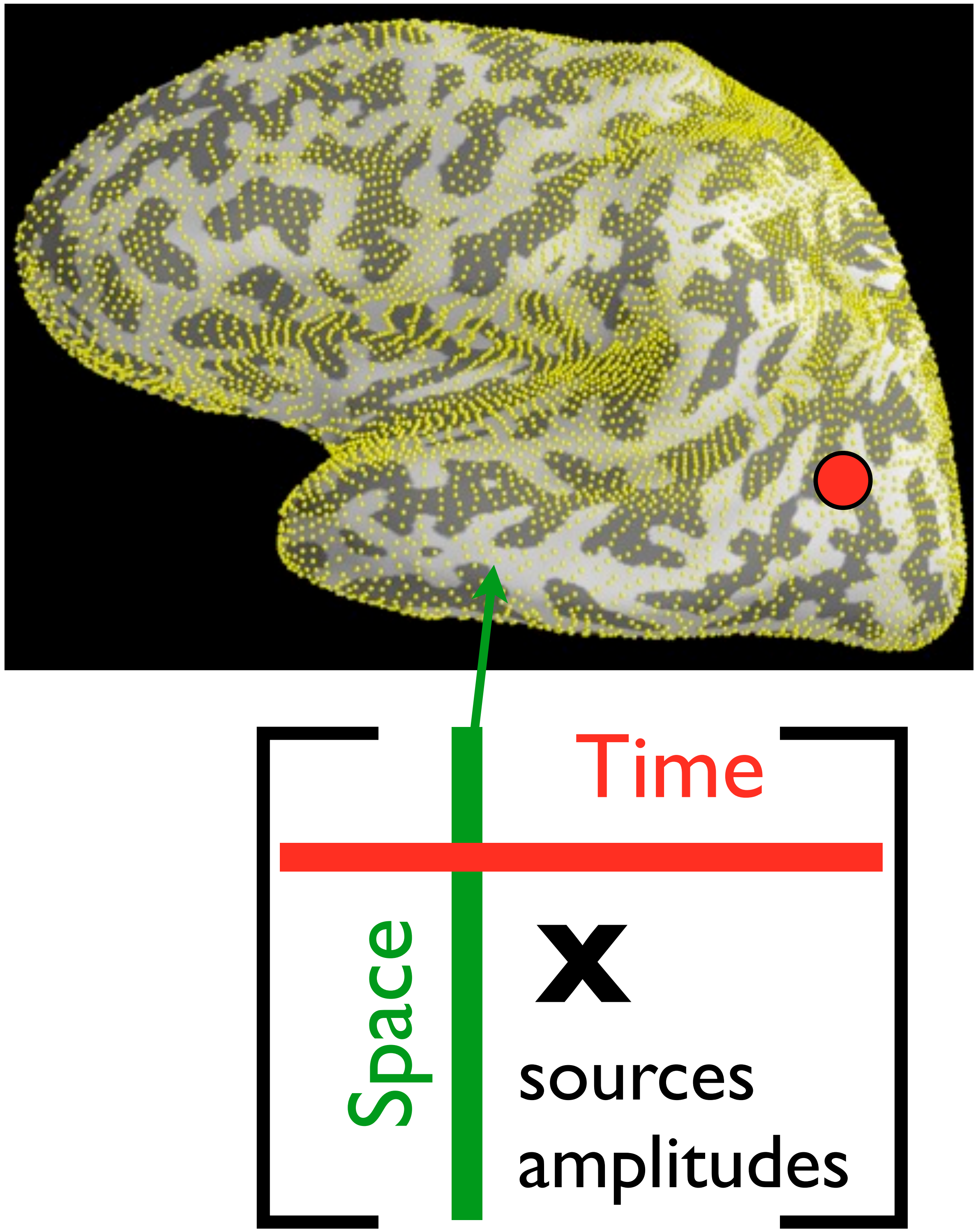
Position 5000 candidate sources over each hemisphere (e.g. every 5mm)



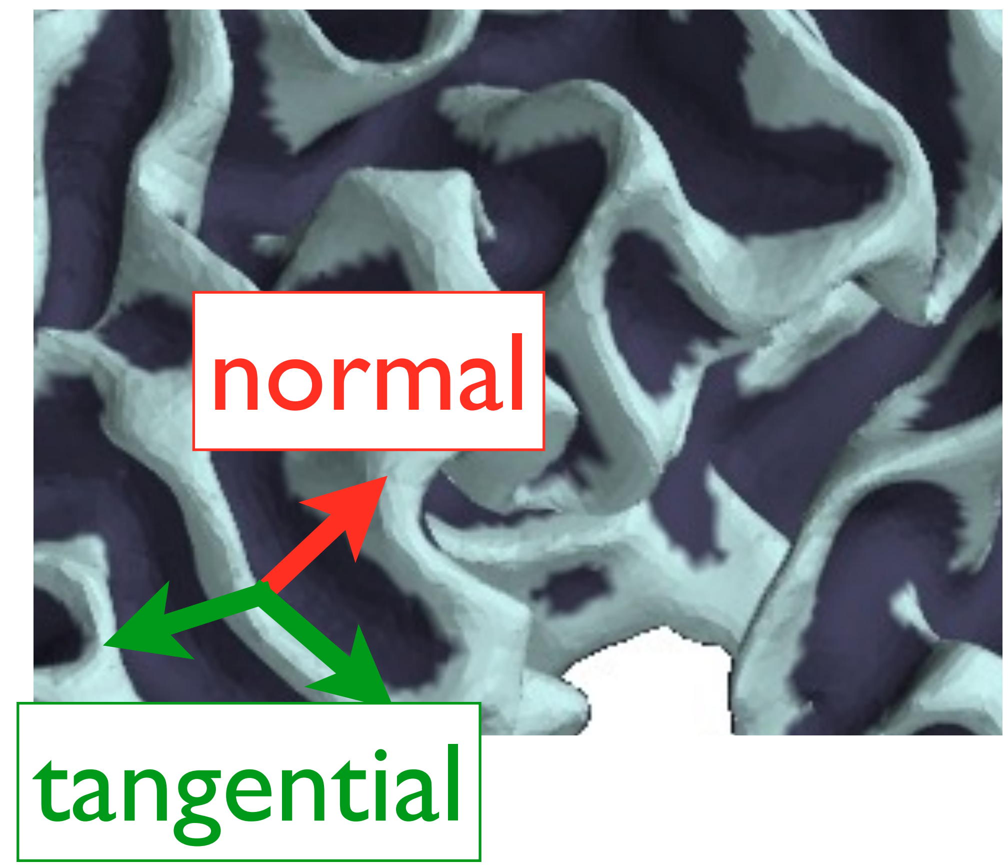


Scalar field defined over time

Distributed model



Or vector field (3 values per location)

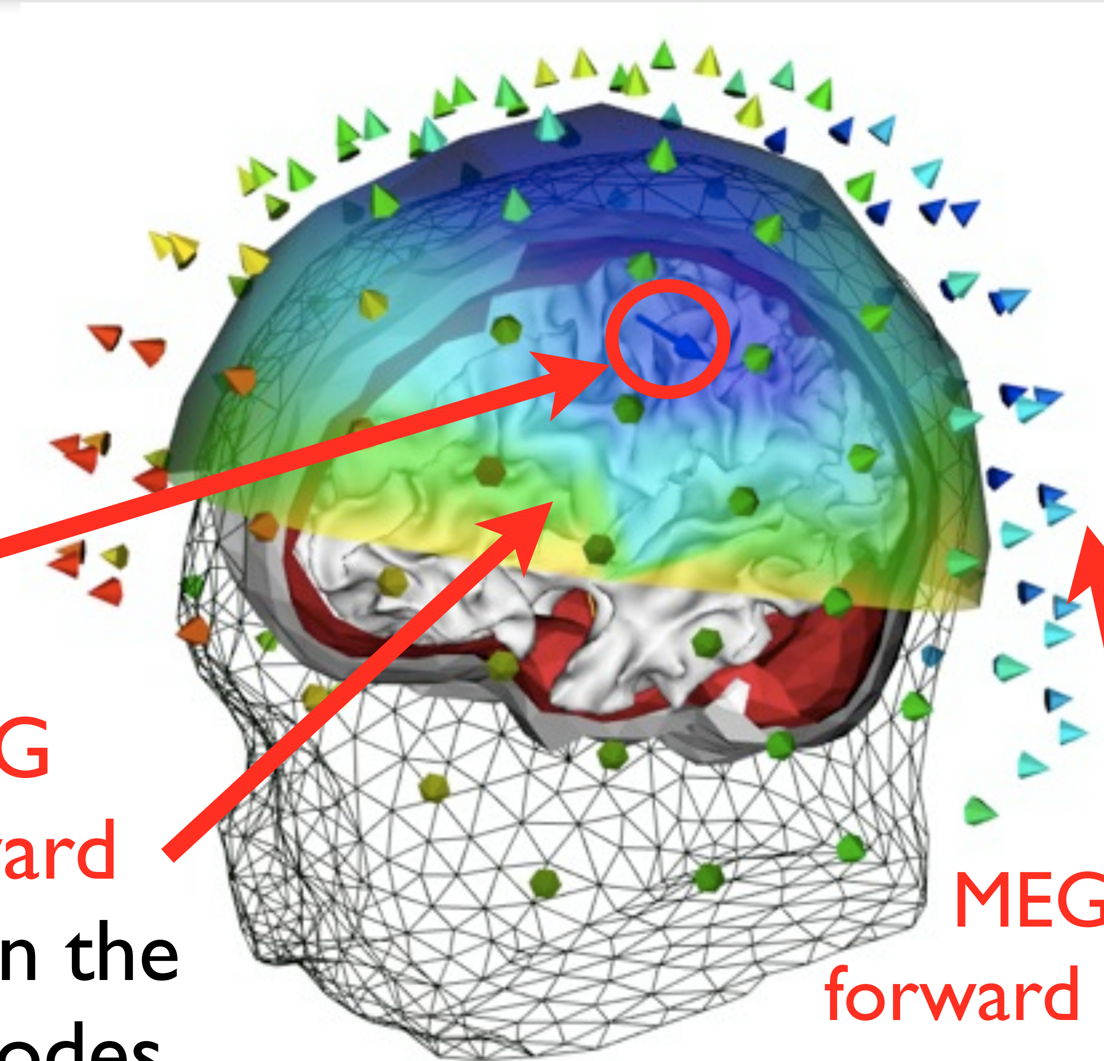


Distributed models

Dipoles sampled over the cortical surface extracted by MRI segmentation

[Dale and Sereno 93]

Current generator modeled as a **current dipole** (location, orientation and amplitude)



EEG forward field on the electrodes

MEG forward field on sensors

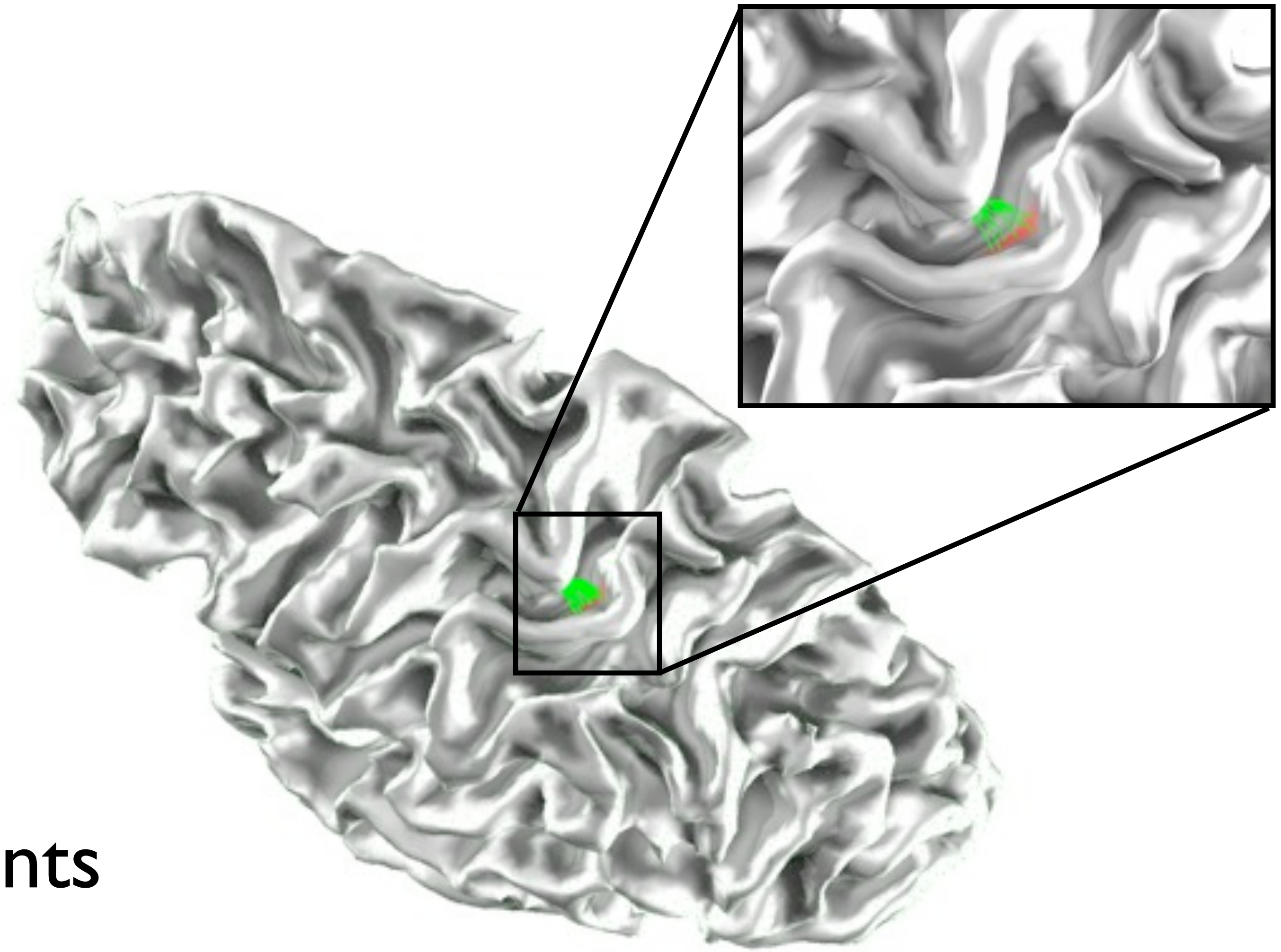
$$G = \left[\quad \right]$$

is the lead field matrix obtained by concatenation of the forward fields

one column = Forward field of one dipole

Distributed source framework

$$\mathbf{M} = \mathbf{G}\mathbf{X} + \mathbf{E}$$



Linear forward model, i.e., M is the sum of the contributions of all the sources (Superposition principle)

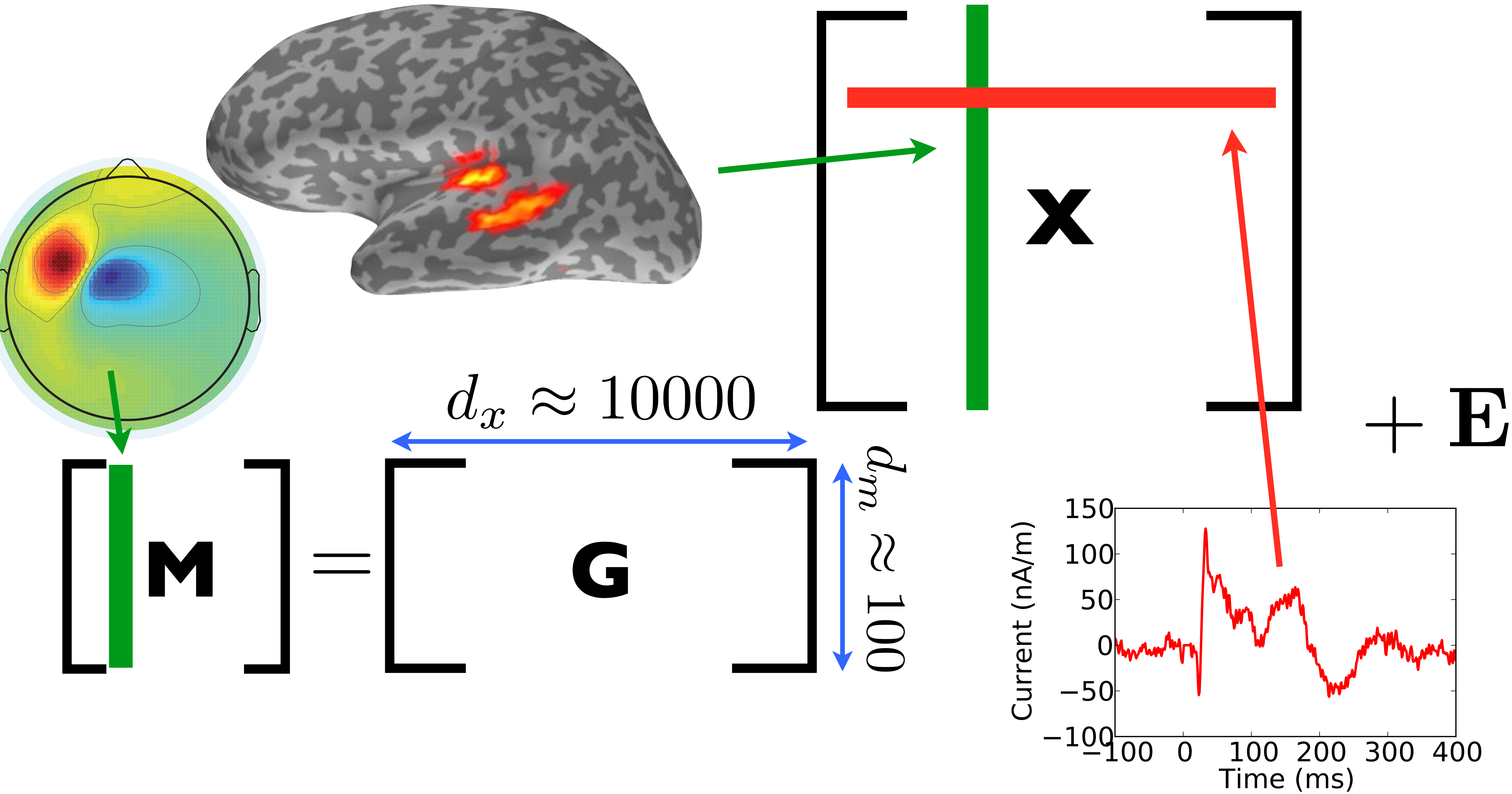
$\mathbf{M} \in \mathbb{R}^{d_m \times d_t}$: M/EEG Measurements

$\mathbf{X} \in \mathbb{R}^{d_x \times d_t}$: Source amplitudes (Unknowns)

$\mathbf{G} \in \mathbb{R}^{d_m \times d_x}$: Leadfield (or Gain) matrix

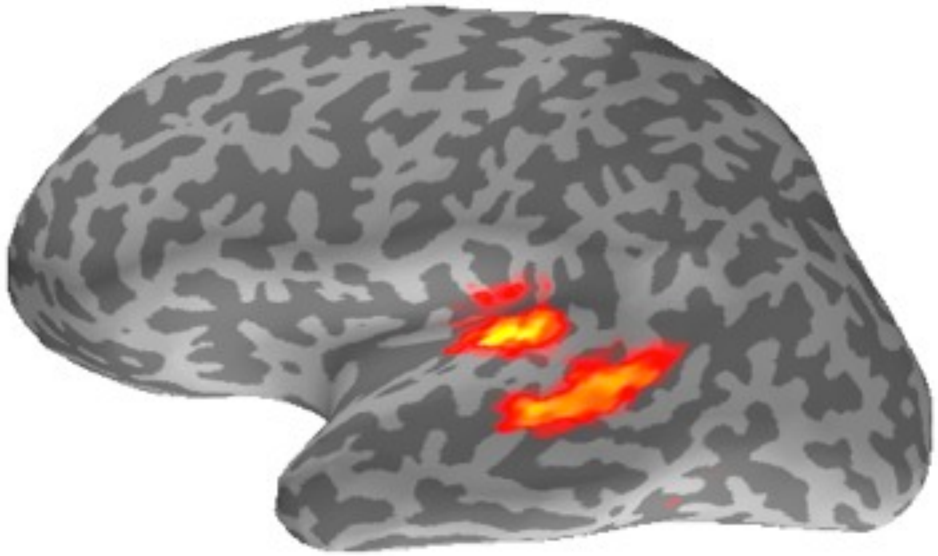
$\mathbf{E} \in \mathbb{R}^{d_m \times d_t}$: additive noise

M = GX+E : An ill-posed problem



Linear problem with more unknowns than the number of equations: it's ill-posed => Use prior

y = Xw+E : An ill-posed problem



Standard statistics notations

w (or β)

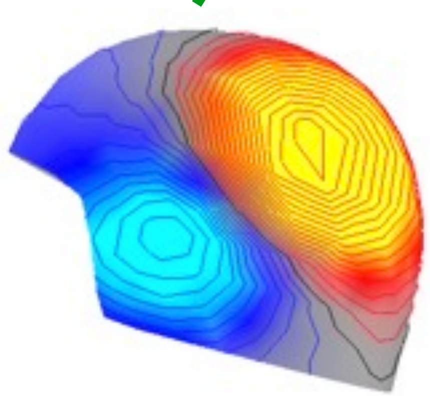
regression coefficients

$d_x \approx 10000$

$d_m \approx 100$

$$\mathbf{y} = \mathbf{X} + \mathbf{E}$$

design matrix



THM: At **each time instant** the M/EEG inverse problem **IS** a **regression** with more variables than observations

Inverse problem framework

Penalized (variational) formulation (with whitened data):

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathbf{M} - \mathbf{G}\mathbf{X}\|_F^2 + \lambda\phi(\mathbf{X}), \lambda > 0$$

Data fit Prior

λ : Trade-off between the data fit and the prior

where $\|\mathbf{A}\|_F^2 = \mathbf{tr}(\mathbf{A}^T\mathbf{A})$

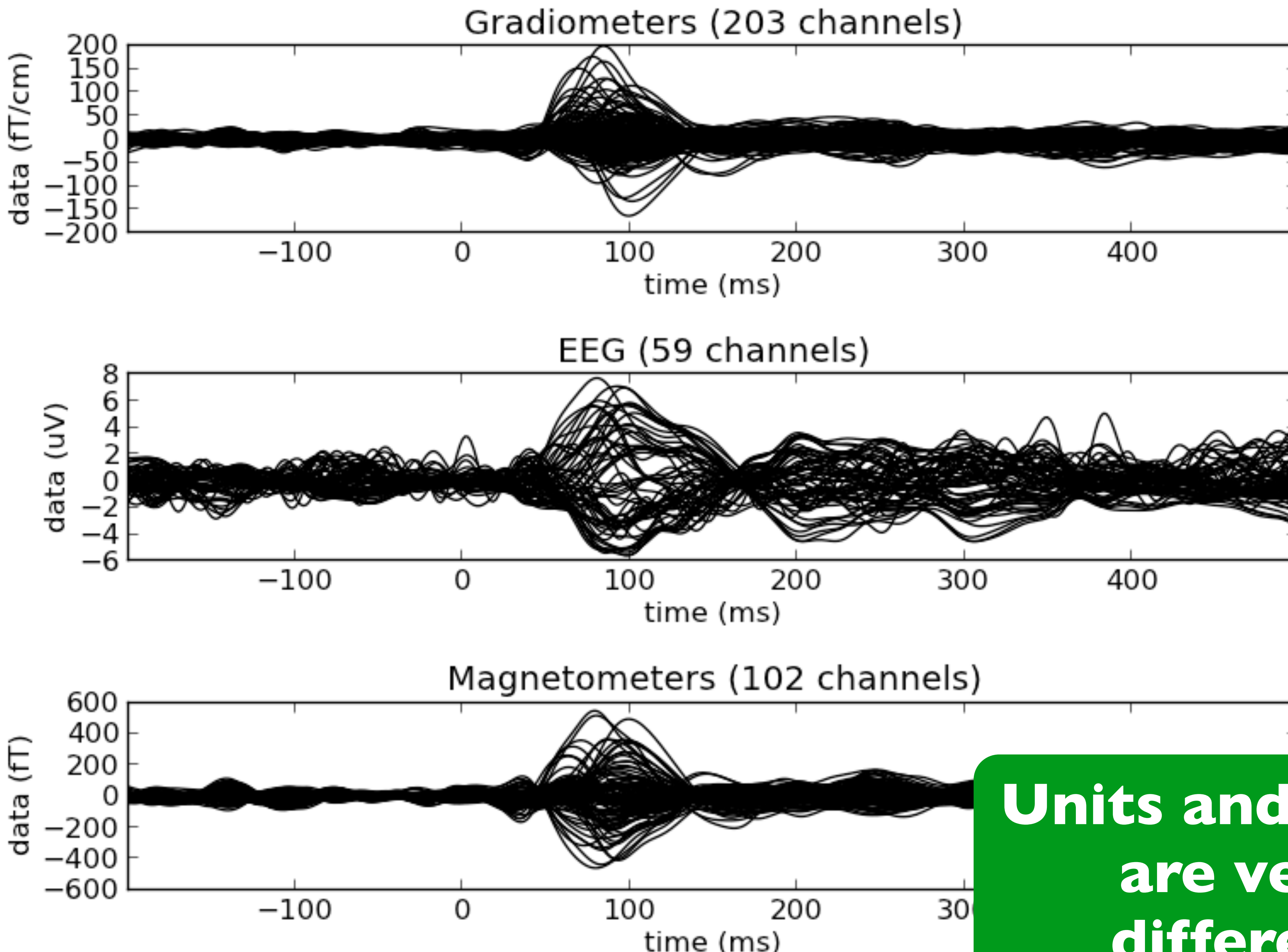
$\phi(\mathbf{X})$ is the prior.

Examples for $\phi(\mathbf{X})$: $\ell_1,\ \ell_2,$ Total-Variation ...

THM: when SNR goes UP λ goes DOWN.

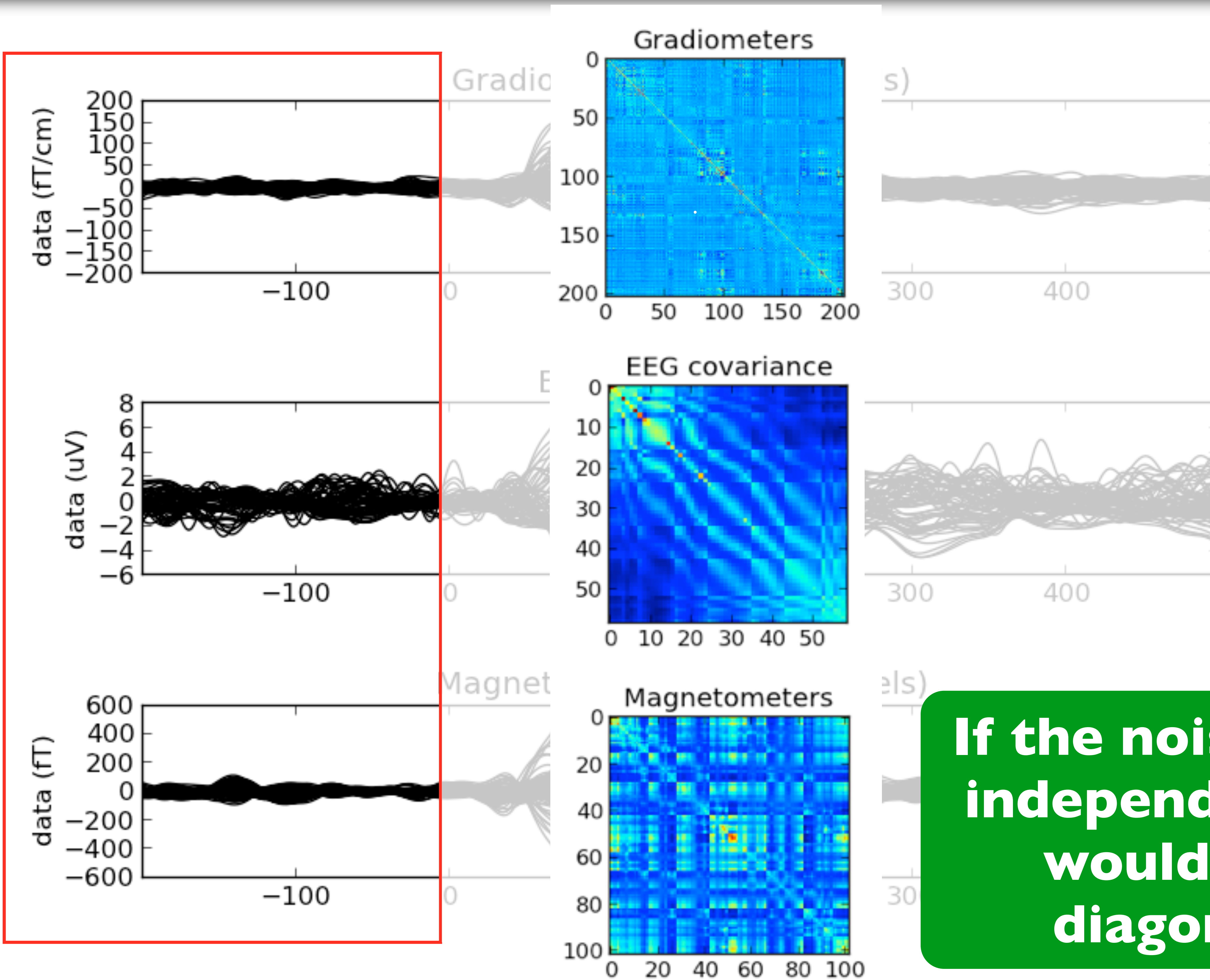
Remark: will only work if all data are on the same scale

Spatial whitening



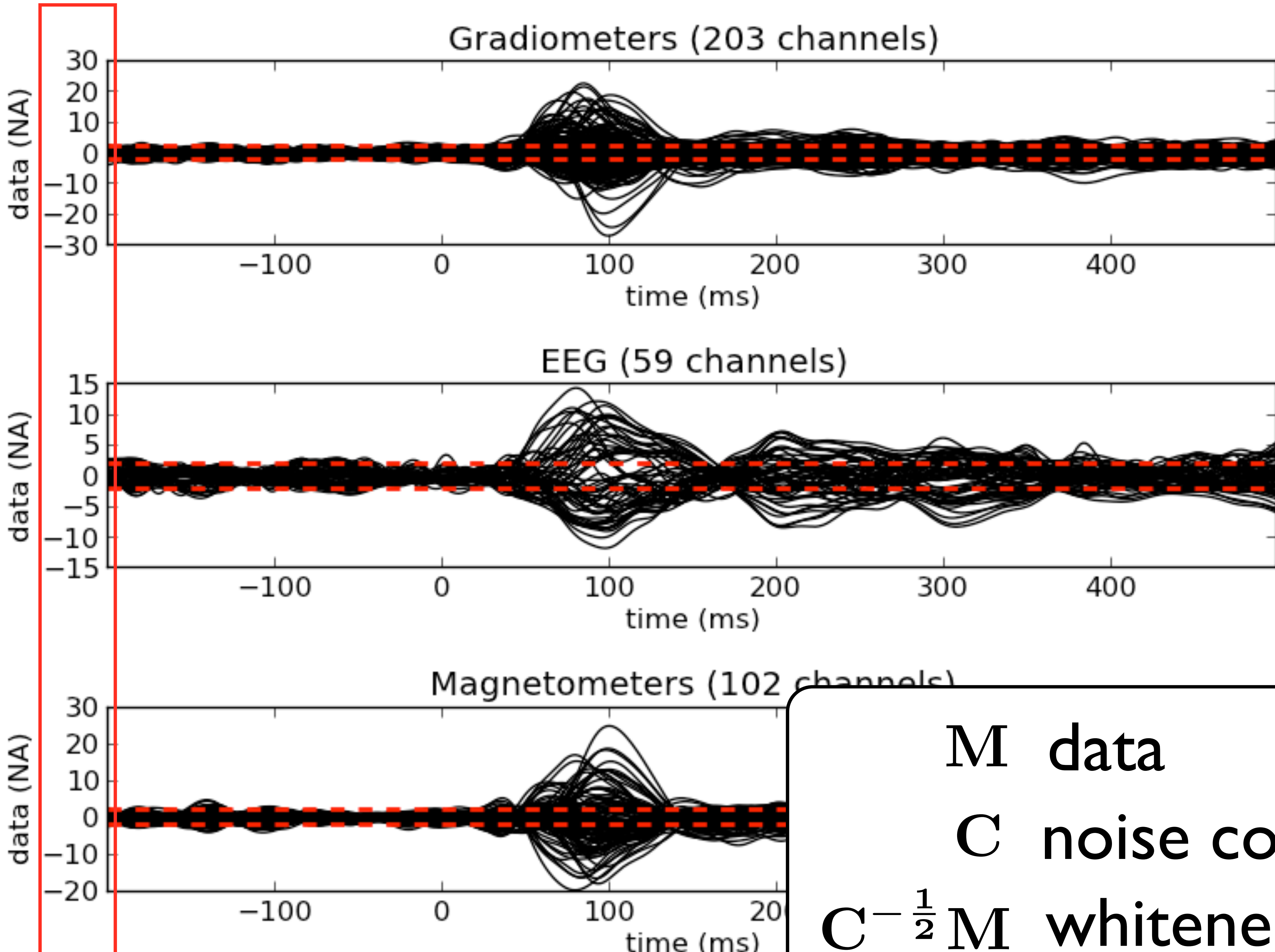
Units and scales are very different

Spatial whitening



If the noise was independent it would be diagonal

Spatial whitening



$\mathbf{M}$ data

$\mathbf{C}$ noise cov.

$\mathbf{C}^{-\frac{1}{2}}\mathbf{M}$ whitened data

L2 a.k.a. Minimum Norm Estimates (MNE)

$$\phi(\mathbf{X}) = \|\mathbf{W}\mathbf{X}\|_F^2 = \sum_{i,j} w_i^2 x_{ij}^2 = \|\mathbf{X}\|_{\mathbf{\Sigma},2}^2$$

$$\mathbf{W}^2 = \mathbf{\Sigma}$$ *source covariance*

Leads to a **closed form solution** (matrix multiplication):

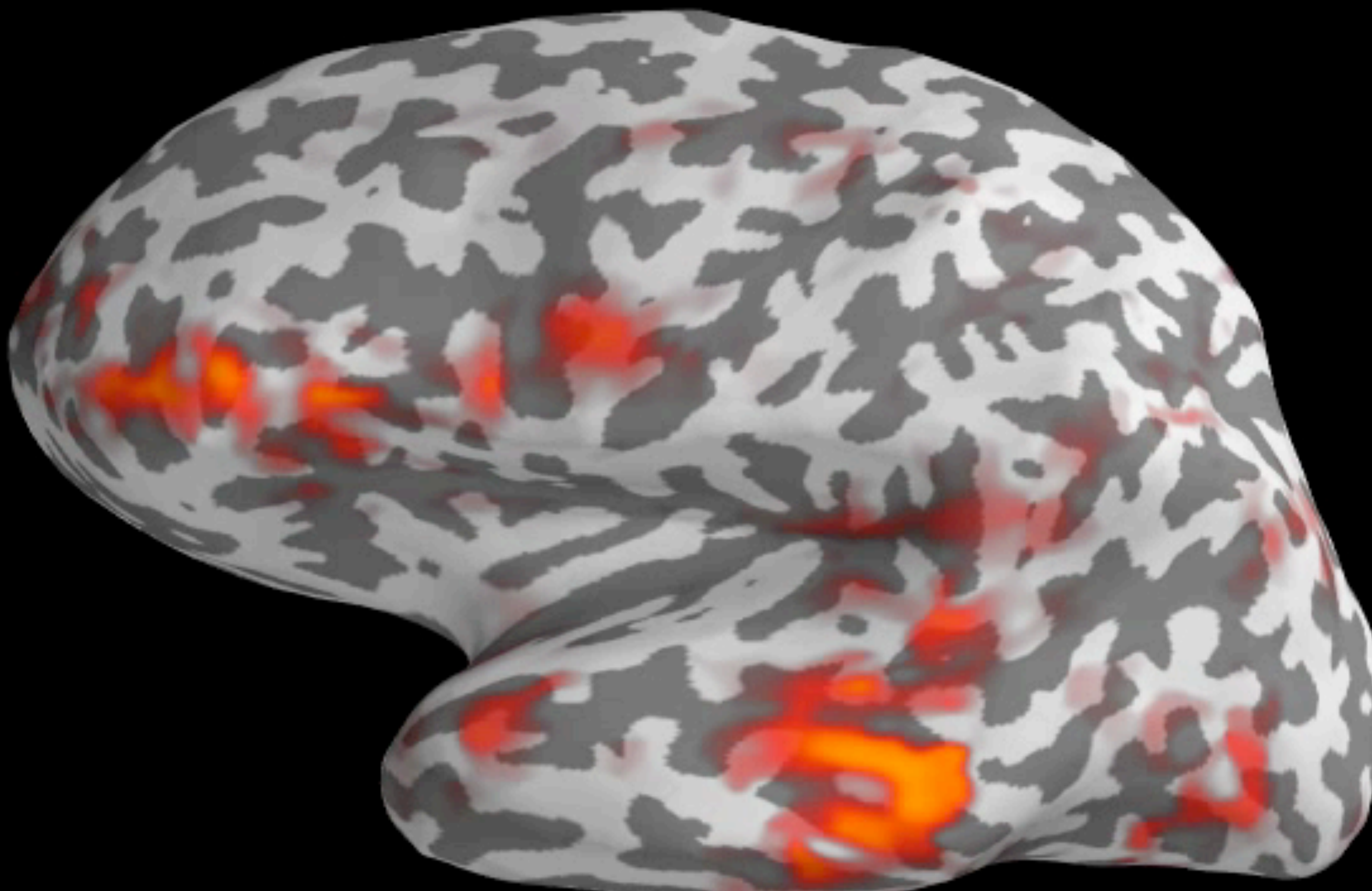
$$\mathbf{X}^* = \underbrace{\mathbf{\Sigma}^{-1}\mathbf{G}^T(\mathbf{G}\mathbf{\Sigma}^{-1}\mathbf{G}^T + \lambda \mathbf{Id})^{-1}}_{\text{Inverse operator}}\mathbf{M}$$

[Tikhonov et al. 77, Wang et al. 92, Hämäläinen et al. 94]

Remarks:

- **Really fast** to compute (SVD of **G**), hence very much used in the field.
- In practice, it's **much more complicated** (whitening data, correcting artifacts, channels with different SNRs, setting λ based on SNR, loose orientation, SNR varies with time...)

THM: A lot of domain knowledge to make it work http://youtu.be/Uxr5Pz7JPrs



time=0.00 ms

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Demo